1888MNRAS..48...96T

No.	Mag.	R.A. 1888·0			δ 1888·0			Authority.	Date for use.
		h	m	s	°	′	″		
7	6·7	13	11	28·86	−10	53	35·0	Gould's Cat.	22, 23
8	7·5		12	56·35	−11	25	30·0	Weisse, Santini	18, 19
9	9·2		14	28	−11	46·7		D.M. −11° 3493	16, 17
10	7		16	12·96	−11	59	32·0	Gould, Paris, 1870	14, 15
11	5		20	48·35	−12	7	27·8	Gould, Yarn. (*i Virginis*)	13, 14
12	9·0		21	32·25	−12	23	25·5	Santini, Lamont	12
13	9·5		21	35	−12	35·5		D.M. −12° 3819	10, 11
14	8·7		24	0·46	−13	25	21·0	Weisse, Santini, Lamont	4, 5
15	7		25	16·70	−12	52	15·0	Gould, Paris, 1870	8, 9
16	9·7		25	47	−13	8·6		D.M. −12° 3834	6, 7
17	8·9		27	26·12	−13	40	13·5	Weisse, Santini	2, 3
18	9·5		31	26	−13	51·6		D.M. −13° 3733	1
19	7·8		32	1·61	−14	1	16·2	Paris, 1855, 1879	Mar. 30, 31
20	9·2		32	26	−14	16·6		D.M. −14° 3763	28, 29
21	8	13	36	52·78	−14	28	7·8	Paris, 1882, 1883	27

Precession for 1888·0 *of the above stars.*

No.	Prec. R.A.	Prec. δ.	No.	Prec. R.A.	Prec. δ.
	s	″		s	″
1	+3·135	−19·32	12	+3·174	−18·79
2	+3·139	−19·26	13	+3·175	−18·79
3	+3·137	−19·24	14	+3·186	−18·72
4	+3·144	−19·15	15	+3·182	−18·68
5	+3·147	−19·12	16	+3·185	−18·66
6	+3·152	−19·10	17	+3·192	−18·61
7	+3·150	−19·08	18	+3·199	−18·47
8	+3·156	−19·04	19	+3·202	−18·46
9	+3·160	−19·00	20	+3·205	−18·44
10	+3·164	−18·95	21	+3·213	−18·29
11	+3·170	−18·82			

Observers are requested to publish their observations as early as possible, so that the final corrections of the assumed elements of the planet's orbit may be obtained.

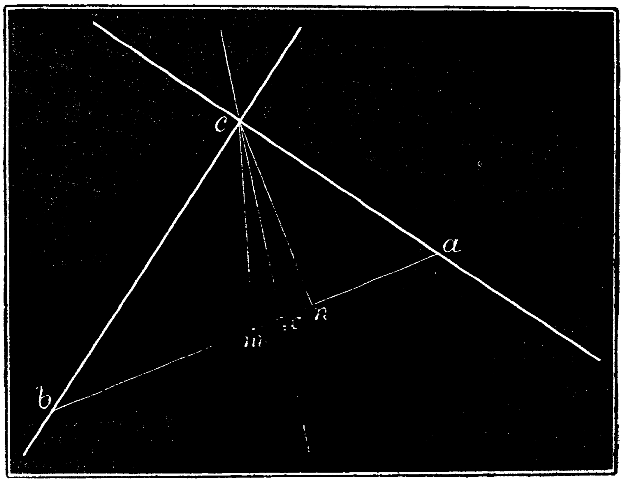
On the Cross Reticule. By G. L. Tupman.

Of the various devices for utilising the diurnal motion of the Earth for micrometric measurements, without the aid of artificial light, a reticule of two straight, stout wires or bars crossing each other at right angles at the centre of the field,

1888MNRAS..48...96T

and *admitting of being accurately oriented*, offers the greatest convenience in practice.

The simple theory of such a reticule may be thus stated:—



Let *ca*, *cb* be two wires inclined to one another at any angle 2γ, forming a simple cross reticule, of which *cx* is the axis lying nearest to an hour-circle, *cn*; let this axis make any angle θ (less than γ) with the hour-circle; let *ab*, perpendicular to *cn*, be the path of a star near the meridian across the reticule, and *m* the middle point of *ab*. Then $acx = \gamma$, $ncx = \theta$, and $mcx = y$ suppose.

The mean of the recorded times of transit at *a* and *b* will be the time of passing the point *m*; but in order to compare two stars in R.A. their transits over the hour-circle *cn* must be compared. Hence the time of describing the distance *nm* must be applied to the mean of the observed times in order to know the instant of passing the point *n*.

Now

$$\sin y = \tan\theta \,.\, \tan^2\gamma$$

$$cn = \frac{\frac{1}{2} ab \,.\, \cos(\gamma+\theta)\cos(\gamma-\theta)}{\sin(\gamma+y)},$$

and

$$mn = cn \,.\, \tan(y+\theta).$$

If now $\gamma = 45°$ and θ be small, we shall have

$$y = \theta$$

$$cn = \tfrac{1}{2} ab$$

$$mn = \tfrac{1}{2} ab \sin 2\theta.$$

These simple expressions show the advantage of using wires at right angles, each inclined *nearly* 45° to the hour-circle, and by-and-by we shall see that some of the small corrections are thereby reduced to their simplest form.

A great advantage of this form of reticule is that the centre

1888MNRAS..48...96T

portion of the field of view, where definition is best, is most used; and the wires being radial are in the best possible position for being sharply defined by the eyepiece.

The bars of my reticule are of watch-spring one-third of a millimeter broad. I had no difficulty in selecting two pieces with perfectly straight edges. The circular aperture of the diaphragm is one inch, over which the bars are held in position by screws. The eyepiece is of the Ramsden form, about $1\frac{1}{4}$ inch focus, the lenses of which are placed a little nearer together than is usual, thus enlarging the available field.

If it were intended to use such a reticule without a position-circle, it would be necessary to place a fibre of silk between the bars and bisecting, as nearly as possible, one pair of the angles. The exact angles which the bars make with the fibre would then have to be carefully determined by observations of selected stars.

The sliding tube which carries the reticule must be fitted with a ring-clamp, on which is a stud or projection fitting cleanly into a corresponding notch in the revolving central portion of the position-circle, so that the connection between the bars and the verniers is rigid, and the zero preserved when the eyepiece is withdrawn.

As soon as the Sun has set,* the telescope should be pointed on rather a bright star near the equator with a small easterly hour-angle. A few minutes will then suffice to adjust the reticule exactly to focus and approximately oriented when the position-circle has any given reading. The ring-clamp can then be tightened, which will permanently fix the bars in focus and in relation to the verniers of the position-circle.

One bar being approximately parallel to the direction of the diurnal motion, bring the star to the centre of the field, and, by means of the slow motion in declination, bisect it with an edge of the said bar, then quickly shifting the telescope so that the star is on the extreme preceding side of the field, turn the position-circle centre until the star is again bisected by the same edge. Allow the star to traverse the field and alter the position of the bar until the star remains bisected along the entire length of the bar.† Then read both verniers of the position-circle.

In this manner the readings of the position-circle are obtained when both edges of both bars are truly oriented. It will be found easy to have a discordance of $3'$ in any one reading, hence this must be done often to ensure obtaining the angle between the bars with the accuracy desirable, say $1'$.

At Harrow, with the telescope on the *East* side of the pillar, the mean reading for bar A was $0° 2'.8$; for bar B $89° 51'.8$.

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* If this be done at night the field of view must be illuminated; but these observations can be made exceedingly well by using the trapezium in the Great Nebula of *Orion*, without artificial illumination.

† If the polar axis of the equatorial be well adjusted, these observations can be more quickly made by swinging backwards and forwards in R.A. with the slow motion.

1888MNRAS..48...96T

The position-circle being graduated in the order NfSp, when its setting is 44° 57′·3 (the zero of orientation) the axis of the larger angle, 90° 11′, will coincide with the hour-circle—in other words, the reticule will be correctly oriented with its larger angle placed for measuring differences of declination.

The zero of orientation thus obtained cannot be used with any other pointing of the telescope unless the observer has satisfied himself that flexure produces no sensible effect.

When the instrument used above was reversed to the *West* side of the pillar, it was found that the bar A was correctly oriented when the reading was 0° 14′·5, repeated reversals giving the same result. This indicates a coefficient of flexure of 8′, a quantity much too large to be neglected.* In practice the flexure is got rid of by obtaining the zero of orientation with the telescope pointing nearly in the same direction as during the comparisons for which the zero is required.

If the zero be obtained by swinging the polar axis it will be the *true* zero; but if obtained by allowing the star to traverse the field of its own motion, it will be the *apparent* zero and affected by refraction. It is immaterial which is used in the reductions until the correction for refraction has to be computed; of which more anon.

Reduction of Observations.

Let t_1, t_2 be the sidereal clock times of transit over the middle of the bars, that is the mean of disappearance and reappearance.

T, the mean $= \frac{1}{2}(t_2 + t_1)$.

τ, the half difference $= \frac{1}{2}(t_2 - t_1)$, considered negative if the object be north of the centre.

D, the distance of the middle point of the path described from the centre of the reticule $= 15\,\tau \sin$ P.

P, the N.P.D. of the object; or, if used without a suffix, the mean N.P.D. of the two objects.

ΔR.A., ΔN.P.D. the differences of R.A. and N.P.D., and let it be supposed throughout that the southern object is the known, and it is required to find the place of the northern object—the objects being indicated by the suffixes N and S.

Then the approximate

$$\Delta\,\text{R.A.} = \text{T}_{\text{N}} - \text{T}$$

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* Bessel, *Astr. Untersuchungen*, i. 14 and ii. 5.

1888MNRAS..48...96T

and approximate

$$\begin{aligned}\Delta\ \text{N.P.D.} &= D_N - D_S \\ &= 15\,(\tau_N - \tau\ )\sin P\end{aligned}$$

unless too near the pole.

If R be the actual setting of the position-circle (supposed to be graduated in the direction N*f*S*p*), and Z the zero of orientation (either true or apparent), the error of orientation (θ) is obtained with its correct sign from

$$\theta = R - Z.$$

Then the time of describing *mn* (of the figure) is

$$2\tau \sin \theta,$$

and for the N. star the

$$\text{corrected } T_N = T_N + 2\tau_N \sin \theta,$$

and for the S. star the

$$\text{corrected } T_S = T_S + 2\tau_S \sin \theta,$$

and, hence the correction required to $\triangle$R.A. for defective orientation is

$$+\,2(\tau_N - \tau_S)\sin \theta.$$

If $90° + \eta$ be the N. and S. angle of the reticule, the correction required to $\triangle$N.P.D. is

$$-\Delta\ \text{N.P.D.}\,.\,\sin \eta,$$

η being constant, a small table may be formed of this correction.

If one of the objects have a proper motion da and dD in R.A. and N.P.D. in 1^s of time, the correction required to $\triangle$N.P.D. is

$$-\,15\tau\,.\,da\,.\,\sin P,$$

and the correction required to $\triangle$R.A. is

$$+\,\frac{\tau\,.\,dD}{15\,(1-\Delta a)\sin P}$$

where τ has the value proper to the object, and is negative if on north side.

The curvature of path will generally be sensible; the correction required to $\triangle$N.P.D. on this account is

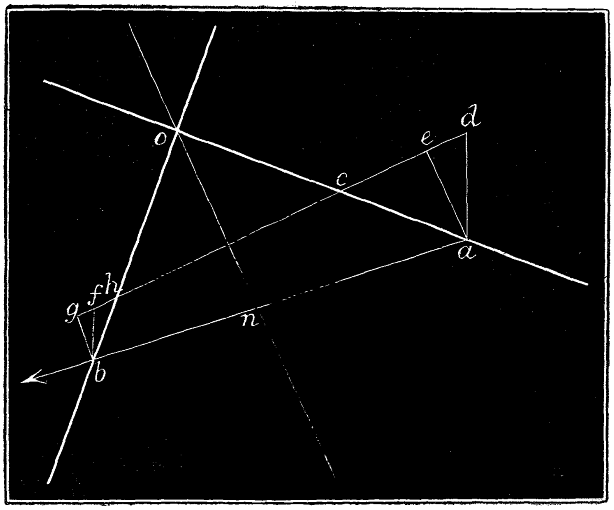
$$-\,[6\,.\,7367]\,(\tau_N + \tau_S)(\tau_N - \tau_S)\sin P\,.\,\cos P.$$

Correction for Refraction.

Let the stars be east of the meridian; *on* the axis of the reticule exactly oriented; *ao*, *bo* the bars; *ab* the observed path of a star in the inverted field, as affected by refraction; *ad* and

1888MNRAS..48...96T

bf, perpendicular to the horizon, the refraction at *a* and *b* respectively. Join *df* and make *ae*, *bg* perpendicular to *dg*, and therefore parallel to *on*.



Then if there were no refraction the star would transit the path *dchg* perpendicular to *on*. *de*, *fg* are the refractions in R.A. which we will call a_1, a_2; *ae*, *bg* are the refractions in N.P.D. which we will call d_1, d_2. In the sense of reducing apparent to true place, the refractions in R.A. are positive when east of the meridian; the refractions in N.P.D. are always positive in the northern hemisphere except in a limited area between the pole and zenith.

Had there been no refraction the star would have been seen at *d* at the instant (t_1) when it was at *a*, and it would have arrived at the point *c* later than t_1 by the time of describing

$$dc\ (=de+ec)=a_1+\frac{d_1}{15\sin\mathrm{P}}.$$

Similarly the star would have been seen at *h* earlier than at *b* by the time of describing

$$hf\ (=hg-fg)=-a_2+\frac{d_2}{15\sin\mathrm{P}}.$$

Therefore

$$\text{corrected } t_1=t_1+a_1+\frac{d_1}{15\sin\mathrm{P}};$$

$$\text{corrected } t_2=t_2+a_2-\frac{d_2}{15\sin\mathrm{P}}.$$

Hence

$$\text{corrected T}=\tfrac{1}{2}(t_2+t_1)+\tfrac{1}{2}(a_2+a_1)-\tfrac{1}{2}(d_2-d_1)\frac{1}{15\sin\mathrm{P}},$$

and

$$\text{corrected } \tau=\tfrac{1}{2}(t_2-t_1)+\tfrac{1}{2}(a_2-a_1)-\tfrac{1}{2}(d_2+d_1)\frac{1}{15\sin\mathrm{P}}.$$

K

1888MNRAS..48...96T

Putting now a and d for the refractions at the middle point of the path, δa, δd, the increase of these quantities respectively in two minutes of time (to be explained further on), we have, assuming the refractions to vary symmetrically over the portion of the field utilised,

$$\text{corrected } \mathrm{T} = \mathrm{T} + a - \frac{\tau}{120} \cdot \frac{\delta d}{15 \sin \mathrm{P}};$$

$$\text{corrected } \tau = \tau - \frac{d}{15 \sin \mathrm{P}} + \frac{\tau}{120} \delta a,$$

and therefore the correction to ΔR.A. is

$$+(a_\mathrm{N} - a_\mathrm{S}) - \frac{\tau_\mathrm{N} - \tau_\mathrm{S}}{120} \cdot \frac{\delta d}{15 \sin \mathrm{P}},$$

and the correction to ΔN.P.D. is

$$-(d_\mathrm{N} - d_\mathrm{S}) + \frac{\tau_\mathrm{N} - \tau_\mathrm{S}}{120} \delta a \,.\, 15 \sin \mathrm{P}.$$

With proper attention to the signs of these quantities the above expressions will be correct under all circumstances, within the limits of the assumption that the change of refraction is proportional to the time in all parts of the field, an assumption that can hardly ever introduce sensible error.

At all observatories auxiliary tables are used for facilitating the computation of refraction for extra-meridional observations. The most simple are thus formed:—

Let ZM be the perpendicular from the apparent zenith Z on the hour-circle passing through the star, and PM the arc included between the pole and the foot of the perpendicular; then

$$\sin \mathrm{ZM} = \sin \text{hour-angle} \times \sin \text{colat.}$$

$$\tan \mathrm{PM} = \cos \text{hour-angle} \times \tan \text{colat.}$$

The angle PM, in degrees and minutes, and the log. tan ZM are tabulated with argument every two minutes of hour-angle; the calculations of the refractions in R.A. and N.P.D. are then readily made from

$$\text{Mean refraction in R.A. in seconds of time} = \frac{[0{\cdot}5836] \tan \mathrm{ZM}}{\sin \text{N.P.D.} \,.\, \cos (\text{N.P.D.} - \mathrm{PM})}.$$

$$\text{Mean refraction in N.P.D. in seconds of arc} = [1{\cdot}7597] \tan (\text{N.P.D.} - \mathrm{PM}).$$

If desired, these refractions can in a moment be corrected for pressure and temperature of the atmosphere.

The refractions are to be computed for the common hour-angle corresponding to the time, T, and the N.P. distances of either object, which give the four quantities a_N, d_N, a_S, d_S. Then adding two minutes to the time, T, the refractions are again computed for one of the objects, and hence are obtained δa and δd.

It is to be noted that the quantities a and d are common to

1888MNRAS..48...96T

all differential work, being the refractions depending on N.P.D. The second terms depend upon the variation of the refractions with hour-angle, and are reduced to their simplest form because the bars are inclined 45° to the hour-circle.

If the *apparent* zero of orientation has been employed in the reductions, the second term of the refraction correction in R.A. disappears because the apparent path of the stars is perpendicular to the axis of the reticule.

As an example I will quote an extreme case that occurred at Harrow when observing the Comet *Barnard*, on 1886, April 8.

	h	m	s		°	′
Local sid. time	10	27	7	Colat.	38	25·2
☄ approx. R.A.	1	48	34	N.P.D.	56	14·7
✱ " "	1	47	3	"	56	4·1

Approx. Δ R.A. + 1 30·94, Δ N.P.D. − 10′ 28″·9.

The mean refractions are:—

	h	m		s	″
At	10	27	☄	−20·72	+469·5
	"		✱	−20·28	+458·5
	10	29	✱	−20·77	+474·0

Whence

$$\delta a = -0^{s}{\cdot}49 \quad \delta d = +15''{\cdot}5.$$

The mean

$$\tau_{N} - \tau_{S} = -50^{s}{\cdot}6,$$

and, therefore, attending to the signs and remarking that the comet is the *southern* object, we have the corrections for refraction:—

	s	″
First term	−0·44	+11·0
Second term	+0·52	− 2·6
	+ ·08	+ 8·4

If the reticule had been oriented by the apparent motion of the stars the corrections would have been

$-0^{s}{\cdot}44 \qquad +8''{\cdot}4.$

Harrow: 1888, *January.*

1888MNRAS..48...96T

Note on the Definition of Reflecting Telescopes and on the Images of Bright Stars on Photographic Plates. By Lieut.-General J. F. Tennant, R.E., F.R.S.

It has frequently been said that the images of stars are seen in reflecting telescopes without any diffraction rings surrounding them. This statement has always surprised me, because it is accompanied by the statement that the discs are very small compared with those in reflectors. The theoretical investigation which accounts for the rings assumes that the optical rays are rigorously convergent, and this is more true for a well-figured speculum than for an object-glass.

Recently, in looking over a translation of Foucault's *mémoire* on "The Construction of Silvered Glass Specula," I was struck by his statement that he always saw "traces of the rings," and I wrote to M. Henry, who, I believe, follows Foucault's methods almost entirely, inquiring what his experience was. M. Henry kindly informs me that not only does he always see rings with his reflectors, but that they are, in fact, easier to see than with a refractor. In his article on Light I find that Sir J. Herschel states that the rings surrounding star images were discovered by Sir W. Herschel, but he says that they are less easily seen than with a refractor.

I have since calculated the places and brilliancy of the rings to be seen in a telescope with an obstructing disc one-third of the diameter of the aperture, centrally placed, and the result has somewhat surprised me.

Calling e the aperture of the telescope in English inches and s the angular diameter of the ring in seconds, Sir George Airy gives for an open aperture the following values, the unit of brilliancy being the centre of the images:—

es	2·76	3·70	5·16	6·09
Brilliancy	0	$\frac{1}{57}$	0	$\frac{1}{240}$

With one-third stopped centrally as above, which would nearly represent a reflector giving a field of 2° diameter, I find—

es	2·47	3·54	5·44	6·22
Brilliancy	0	$\frac{1}{18}$	0	$\frac{1}{712}$

It would follow that while the central image is smaller and also the first dark ring, the first bright ring would be far more conspicuous than in a refractor, both by its width and brilliancy, compared with the central image.

The place of the third dark ring would be very little inside the corresponding place in a refractor, and so would that of the